Point support of CLT

Analysis sample and theoretical background

Date: 10.01.2014

rethink.

storaenso

Point support of CLT

ANALYSIS SAMPLE 01/2014

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Table of content

1. General

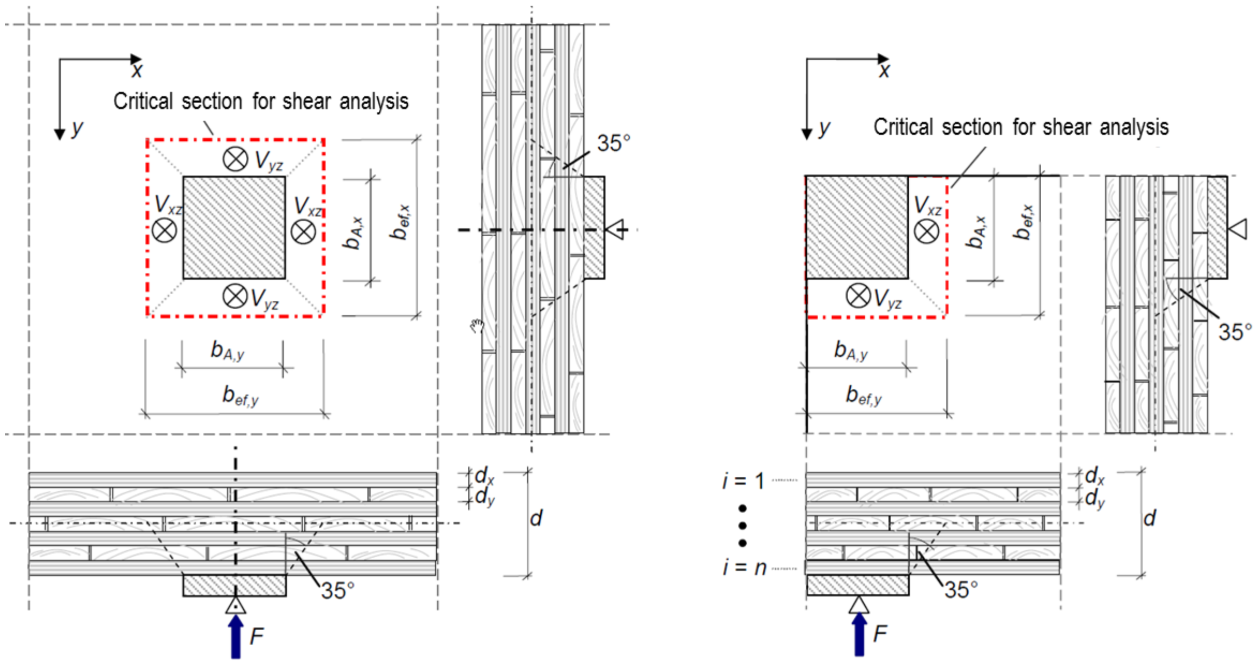
This document shall explain point support situations on CLT in a practical way, by means of an analysis sample. CLT is a load bearing shell element (or slab) that is capable of spanning in 2 directions. This capability makes point supports very attractive. A typical situation would be a CLT slab that is directly supported by a column, without a beam as load distributing element in between. Such structures can create very efficient and elegant solutions.
There are not many research documents related to shear stress at point supports on CLT slabs yet. The most relevant information could be found in the doctoral thesis by Dipl.-Ing. Peter Mestek (Mestek, 2011). Additionally to the shear analysis, the bearing pressure needs to be analyzed.

2. Analytical equations and theory

The design of point supports includes the shear analysis and the bearing pressure analysis (cross grain bearing) on the CLT.

2.1. Shear analysis (Mestek, 2011)

The critical design criterion for point supports on CLT is the rolling shear analysis. The images below indicate the relevant section for the shear design (Mestek, 2011):
left side: situation in the center of a slab; right side: situation at a corner of the slab



The relevant design equations for the cases described above can be found in the table below (Mestek, 2011):

System		Point support or point load in the center of a CLT slab	Point support or point load in the corner of a CLT slab
Shear force	x-direction	$V_{xz} \approx 0{,}33 \cdot n^{-0{,}1} \cdot F$	$V_{xz} \approx 0{,}67 \cdot n^{-0{,}1} \cdot F$
	y-direction	$V_{yz} \approx 0{,}5 \cdot F - V_{xz}$	$V_{yz} \approx F - V_{xz}$
Tributary width	x-direction	$b_{ef,x} = b_{A,x} + d \cdot \tan 35°$	$b_{ef,x} = b_{A,x} + d/2 \cdot \tan 35°$
	y-direction	$b_{ef,y} = b_{ef,x}$	$b_{ef,y} = b_{ef,x}$
Rolling shear stress	x-direction	$\tau_{R,xz} = \dfrac{V_{xz} / b_{ef,x}}{k_{R,x} \cdot (d_x + d_y)}$	$\tau_{R,xz} = \dfrac{V_{xz} / b_{ef,x}}{k_{R,x} \cdot (d_x + d_y)} \cdot k_A$
	y-direction	$\tau_{R,yz} = \dfrac{V_{yz} / b_{ef,y}}{k_{R,y} \cdot (d_x + d_y)}$	$\tau_{R,yz} = \dfrac{V_{yz} / b_{ef,y}}{k_{R,y} \cdot (d_x + d_y)} \cdot k_A$

Number of layers	5	7	9	11
$k_{R,x}$	2,00	2,50	3,33	3,89
$k_{R,y}$	1,00	2,00	2,50	3,33

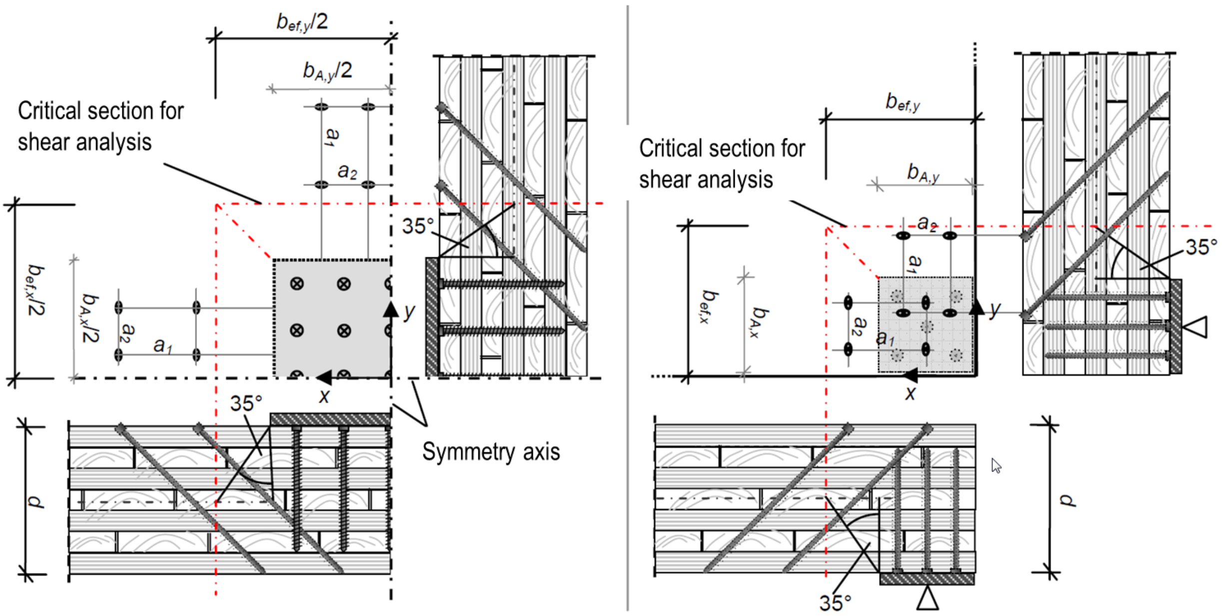
$b_{A,x}/d$ or $b_{A,y}/d$	≤ 1,0	≤ 1,5	≤ 2,0
factor k_A	1,35	1,50	1,65

Certain limitations apply to the model, described above:

- Support surface is square
- The thickness of the layers in principal direction d_x and thickness of the layers in cross direction d_y are equal
- The layering of the CLT section is symmetrical

2.2. Shear analysis for reinforces supports (Mestek, 2011)

Similar to the reinforcing method for glulam beams, CLT can be reinforced as well, suing fully threaded screws. The images below indicate the relevant section for the shear design (Mestek, 2011):
left side: situation in the center of a slab; right side: situation at a corner of the slab



$$a_{2,ef} = max.\begin{cases} a_2 \\ b_{ef,x}/n_\perp \end{cases}$$

$n_\perp$ is the number of screw rows perpendicular to the related bearing direction

The bearing pressure for reinforced sections includes a component for plain bearing pressure and a component related to the screws. Therefore the bearing pressure shall be analyzed as follows:

$$\sigma_{c,90,k} = \frac{F_k}{b_{ef,x} \cdot b_{ef,y}} + \frac{R_{ax,k}/\sqrt{2}}{a_1 \cdot a_{2,ef}}$$

For the verification of the rolling shear stress, the rolling shear form an unreinforced CLT slab is being compared to the design rolling shear capacity of the reinforced CLT slab as follows:

$$\tau_{R,d} \leq \frac{k_{mod} \cdot \bar{f}_{R,k}}{\gamma_M}$$

The bearing capacity for the reinforced section shall be:

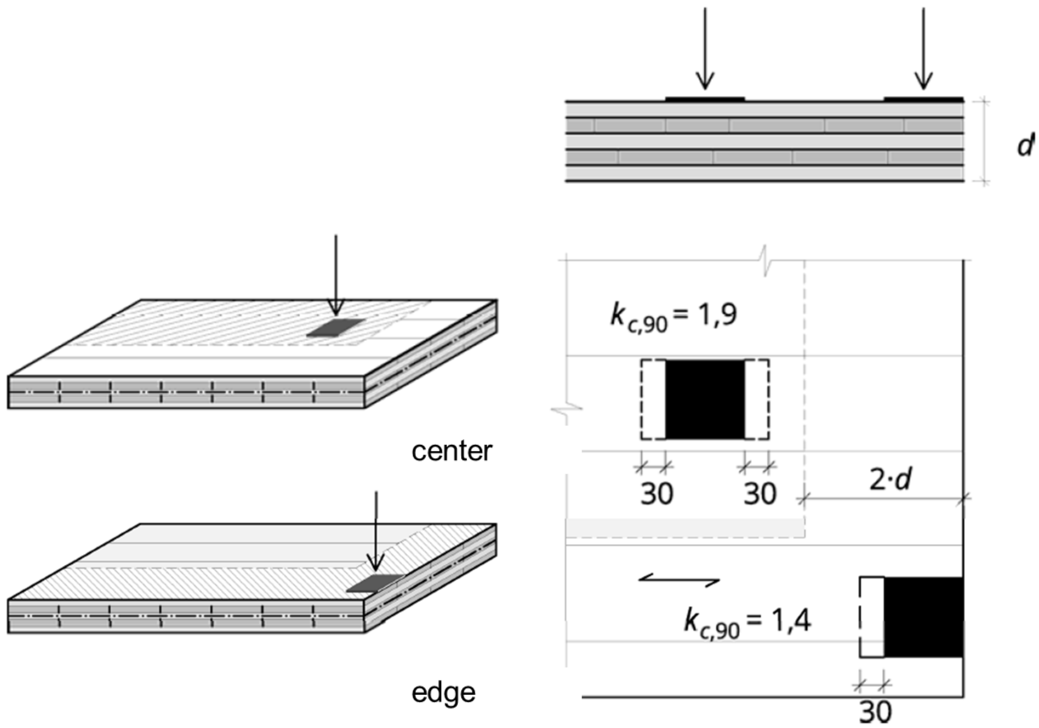
$$\bar{f}_{R.k} = k_{R,90} \cdot f_{R,k} + \frac{R_{ax,k}/\sqrt{2}}{a_1 \cdot a_{2,ef}}$$

$$k_{R,90} = min. \begin{cases} 1 + 0{,}35 \cdot \sigma_{c,90,k} \\ 1{,}20 \end{cases}$$ $\sigma_{c,90,k}$ in N/mm²

2.3. Bearing pressure (Wallner-Novak, 2013)

The design equation for bearing pressure perpendicular to the grain is as follows:

$$\sigma_{c,90,d} = \frac{P_{90,d}}{k_{c,90} \cdot A_{eff}} \leq f_{c,90,d} = \frac{f_{c,90,k} \cdot k_{mod}}{\gamma_M}$$



Sources:
The effective area A_{eff} is based on the theory by Prof. Dr.-Ing. Hans Joachim Blass (Blass & Görlacher).
The k_{c90} values for CLT are according to (Bogensperger, Augustin, & Schickhofer).

2.4. Bearing pressure for reinforced sections

In order to increase the bearing pressure capacity of a CLT section, reinforcement with fully threaded screws can be used. The following design procedure was taken from the European technical approval for Rothoblaas, ETA-11/0030 (Rothoblaas, 2012).

Compression reinforcement

"VGS" and "VGZ" screws with a full thread may be used for reinforcement of timber members with compression stresses at an angle α to the grain of $45° < \alpha < 90°$. The compression force must be evenly distributed over all screws. The characteristic load-carrying capacity for a contact area with screws with a full thread at an angle α to the grain of $45° < \alpha < 90°$ shall be calculated from:

$$F_{90,Rk} = min\begin{cases} k_{c,90} \cdot A_{eff} \cdot f_{c,90,k} + n \cdot min\{F_{ax,Rk}; F_{ki,Rk}\} \\ l_{ef,3} \cdot l_{ef,2} \cdot f_{c,90,k} \end{cases}$$

Where

$F_{90,Rk}$	load-carrying capacity of reinforced contact area [N]
$k_{c,90}$	factor for compression perpendicular to the grain according to EN 1995-1-1:2008, 6.1.5
B	bearing width [mm]
A_{eff}	Effective bearing contact area according to item 2.3 above
$l_{ef,1}$	effective length of contact area according to EN 1995-1-1:2008, 6.1.5 [mm]
$f_{c,90,k}$	characteristic compressive strength perpendicular to the grain [N/mm²]
n	number of reinforcement screws, $n = n_0 \cdot n_{90}$
n_0	number of reinforcement screws arranged in a row parallel to the grain
n_{90}	number of reinforcement screws arranged in a row perpendicular to the grain
$F_{ax,Rk}$	characteristic axial withdrawal capacity [N]
$F_{ki,Rk}$	characteristic buckling capacity [N]
$l_{ef,2}$	effective distribution length in the plane at the screw tips in x-direction[mm] $l_{ef,2} = l_{ef} + (n_0 - 1) \cdot a_1 + min(l_{ef}; a_{1,c})$ for reinforced end-bearings [mm] $l_{ef,2} = 2 \cdot l_{ef} + (n_0 - 1) \cdot a_1$ for reinforced centerbearings [mm]
$l_{ef,3}$	effective distribution length in the plane at the screw tips in y-direction[mm] $l_{ef,2} = l_{ef} + (n_0 - 1) \cdot a_1 + min(l_{ef}; a_{1,c})$ for reinforced end-bearings [mm] $l_{ef,2} = 2 \cdot l_{ef} + (n_0 - 1) \cdot a_1$ for reinforced centerbearings [mm]
l_{ef}	point side penetration length [mm]
a_1	spacing parallel to the grain [mm]
$a_{1,c}$	end distance [mm]

Reinforcing screws for compression shall be arranged according to Annex C of ETA-11/0030. Reinforcing screws for wood-based panels are not covered by this European Technical Approval. The characteristic buckling capacity $F_{ki,Rk}$ shall be calculated from: $F_{ki,Rk} = \kappa_c \cdot N_{pl,k}$ [N]

where

$$\kappa_c = \begin{cases} 1 & \text{for } \bar{\lambda}_k \le 0,2 \\ \dfrac{1}{k + \sqrt{k^2 - \bar{\lambda}_k^2}} & \text{for } \bar{\lambda}_k > 0,2 \end{cases}$$

$$k = 0,5 \cdot \left[1 + 0,49 \cdot (\bar{\lambda}_k - 0,2) + \bar{\lambda}_k^2\right]$$

The relative slenderness ratio shall be calculated from:

$$\bar{\lambda}_k = \sqrt{\frac{N_{pl,k}}{N_{ki,k}}} \quad [-]$$

where

$$N_{pl,k} = \pi \cdot \frac{d_1^2}{4} \cdot f_{y,k} \quad [N]$$

is the characteristic value for the axial capacity in case of plastic analysis referred to the inner thread cross section.
Characteristic yield strength of screws from carbon steel: $f_{y,k} = 1000$ [N/mm²]
Characteristic ideal elastic buckling load:

$$N_{ki,k} = \sqrt{c_h \cdot E_S \cdot I_S} \quad [N]$$

Elastic foundation of the screw:

$$c_h = (0,19 + 0,012 \cdot d) \cdot \rho_k \cdot \left(\frac{\alpha}{180°} + 0,5\right) \quad [N/mm^2]$$

Modulus of elasticity: $E_s = 210.000$ [N/mm²]

Second moment of area:

$$I_S = \frac{\pi}{64} \cdot d_1^4 \quad [mm^4]$$

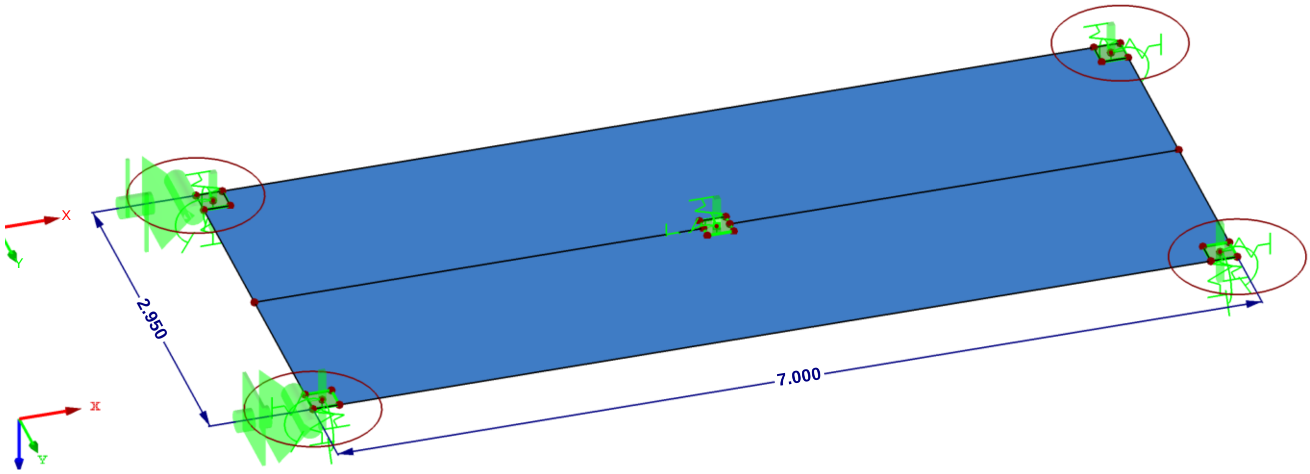
d_1 = inner thread diameter [mm]
Note: When determining design values of the compressive capacity it should be considered that $f_{ax,d}$ is to be calculated using k_{mod} and γ_M for timber according to EN 1995 while $N_{pl,d}$ is calculated using $\gamma_{M,0}$ for steel according to EN 1993.

3. Analysis sample - unreinforced

A CLT floor slab shall be supported by 5 point supports – one at each corner of the CLT slab and one in the center of the slab. The ULS design shall be performed for the given support condition

3.1. Geometry & load assumption

The following image has been taken from the finite element software RFEM by Dlubal.



Geometry & material:

- CLT 200 L5s, pine C24
- Length l = 7,00 m
- Width w = 2,95 m
- End plates of the steel columns are all 200 x 200 mm. These end plates are the supports for the given slab.

Loading:

- Dead load: self-weight of the CLT slab + a surcharge of 1,5 kN/m²
- Live load: 2,00 kN/m²

3.2. Support reactions by FE software

The support reactions have been taken from the finite element analysis software:

Center support:

- DL = 28,59 kN
- LL = 22,87 kN

Design support reaction (governing load combination):

$$P_d = \sum \gamma_{G,J} \cdot G_{k.j} + \gamma_{Q,1} \cdot Q_{k.1} \cdot \sum \gamma_{Q,i} \cdot \psi_{0,i} \cdot Q_{k.i}$$

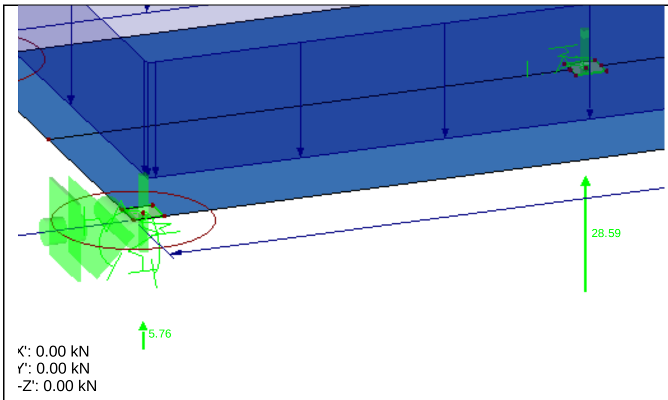
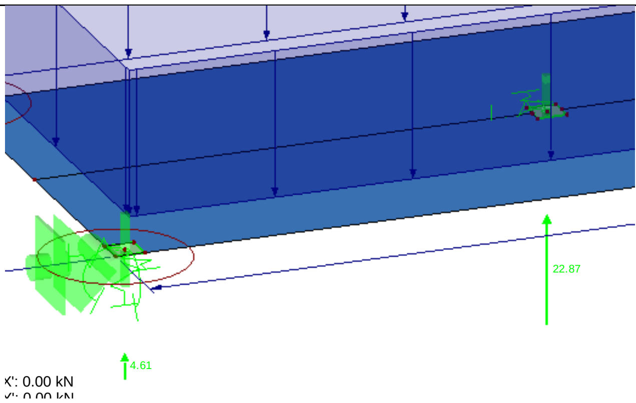
$$P_{center,d} = 1{,}35 \cdot DL + 1{,}50 \cdot LL = 1{,}35 \cdot 28{,}59\ kN + 1{,}50 \cdot 22{,}87\ kN = \mathbf{72,90\ kN}$$

Corner supports:

- DL = 5,76 kN
- LL = 4,16 kN

Design support reaction (governing load combination):

$$P_{corner,d} = 1{,}35 \cdot DL + 1{,}50 \cdot LL = 1{,}35 \cdot 5{,}76\ kN + 1{,}50 \cdot 4{,}61\ kN = \mathbf{14,69\ kN}$$

	
Screenshot RFEM: DL support reaction	Screenshot RFEM: LL support reaction

3.3. Shear design for point support

3.3.1. Center support

$$V_{xz,d} = 0{,}33 \cdot n^{-0{,}1} \cdot P_{center,d} = 0{,}33 \cdot 5^{-0{,}1} \cdot 72{,}90\ kN = 20{,}47\ kN$$

$$V_{yz,d} = 0{,}5 \cdot P_{center,d} - V_{xz,d} = 0{,}5 \cdot 72{,}90 - 20{,}47 = 15{,}987\ kN$$

$$b_{ef,x} = b_{ef,y} = b_{A,x} + d \cdot tan35° = 200\ mm + 200\ mm \cdot tan35° = 340\ mm$$

$$\tau_{R,xz,d} = \frac{V_{xz,d}}{b_{ef,x} \cdot k_{R,x} \cdot (d_x + d_y)} = \frac{20.470\ N}{340\ mm \cdot 2 \cdot (40+40)} = 0{,}37\ N/mm^2 (not\ governing)$$

$$\tau_{R,yz,d} = \frac{V_{yz,d}}{b_{ef,y} \cdot k_{R,y} \cdot (d_x + d_y)} = \frac{15.987\ N}{340\ mm \cdot 1 \cdot (40+40)} = 0{,}58\ N/mm^2 (governing)$$

$$f_{R,k,pine} = min \begin{cases} 1{,}70 \\ 1{,}90 - \dfrac{t_{max,cross\ layer}[mm]}{100} \end{cases} \rightarrow = 1{,}90 - \frac{40\ mm}{100} = 1{,}50\ N/mm^2$$

$$\tau_{R,yz,d} = 0{,}58\ N/mm^2 \leq f_{R,d} = \frac{f_{R,k} \cdot k_{mod}}{\gamma_M} = \frac{1{,}50\ N/mm^2 \cdot 0{,}80}{1{,}30} = 0{,}92\ N/mm^2 \rightarrow o.k. ✓$$

$$Utilization\ rate\ UR = \frac{0{,}58\ N/mm^2}{0{,}92\ N/mm^2} = 0{,}63 \rightarrow 63\%$$

3.3.2. Corner support

$$V_{xz,d} = 0{,}67 \cdot n^{-0{,}1} \cdot P_{corner,d} = 0{,}67 \cdot 5^{-0{,}1} \cdot 14{,}69\ kN = 8{,}38\ kN$$

$$V_{yz,d} = P_{corner,d} - V_{xz,d} = 14{,}69 - 8{,}38 = 6{,}31\ kN$$

$$b_{ef,x} = b_{ef,y} = b_{A,x} + \frac{d \cdot tan35°}{2} = 200\ mm + \frac{200\ mm \cdot tan35°}{2} = 270\ mm$$

$$\tau_{R,xz,d} = \frac{V_{xz,d}}{b_{ef,x} \cdot k_{R,x} \cdot (d_x + d_y)} \cdot k_A = \frac{8.380\ N}{270\ mm \cdot 2 \cdot (40+40)} \cdot 1{,}35 = 0{,}26\ N/mm^2 (not\ governing)$$

$$\tau_{R,yz,d} = \frac{V_{yz,d}}{b_{ef,y} \cdot k_{R,y} \cdot (d_x + d_y)} \cdot k_A = \frac{6.310\ N}{270\ mm \cdot 1 \cdot (40+40)} \cdot 1{,}35 = 0{,}39\ N/mm^2 (governing)$$

$$f_{R,k,pine} = 1{,}50\ N/mm^2 \qquad (see\ above)$$

$$\tau_{R,xz,d} = 0{,}39\ N/mm^2 \leq f_{R,d} = \frac{f_{R,k} \cdot k_{mod}}{\gamma_M} = \frac{1{,}50\ N/mm^2 \cdot 0{,}80}{1{,}30} = 0{,}92\ N/mm^2 \rightarrow o.k. \checkmark$$

$$Utilization\ rate\ \ UR = \frac{0{,}39\ N/mm^2}{0{,}92\ N/mm^2} = 0{,}42\ \rightarrow 42\%$$

3.3.3. Conclusion from shear design

In both cases, at the center support and at the corner supports, the design shear stress does not exceed the design rolling shear strength → o.k.

3.4. Bearing pressure analysis

3.4.1. Center support

$$\sigma_{c,90,d} = \frac{P_{center,d}}{k_{c,90} \cdot A_{eff}} = \frac{72.900\ kN}{1{,}9 \cdot [200\ mm\ \cdot (2 \cdot 30\ mm + 200\ mm)]} = 0{,}74\ N/mm^2$$

$$f_{c,90,d} = \frac{f_{c,90,k} \cdot k_{mod}}{\gamma_M} = \frac{2{,}5\ N/mm^2 \cdot 0{,}80}{1{,}30} = 1{,}54\ N/mm^2$$

$$\sigma_{c,90,d} = 0{,}74\ N/mm^2 \leq f_{c,90,d} = 1{,}54 N/mm^2 \rightarrow o.k. \checkmark$$

$$Utilization\ rate\ \ UR = \frac{0{,}74\ N/mm^2}{1{,}54\ N/mm^2} = 0{,}48\ \rightarrow 48\%$$

3.4.2. Corner support

$$\sigma_{c,90,d} = \frac{P_{corner,d}}{k_{c,90} \cdot A_{eff}} = \frac{14.690\ kN}{1{,}4 \cdot [200\ mm\ \cdot (30\ mm + 200\ mm)]} = 0{,}23\ N/mm^2$$

$$f_{c,90,d} = \frac{f_{c,90,k} \cdot k_{mod}}{\gamma_M} = \frac{2{,}5\ N/mm^2 \cdot 0{,}80}{1{,}30} = 1{,}54\ N/mm^2$$

$$\sigma_{c,90,d} = 0{,}23\ N/mm^2 \leq f_{c,90,d} = 1{,}54 N/mm^2 \rightarrow o.k. \checkmark$$

$$Utilization\ rate\ \ UR = \frac{0{,}23\ N/mm^2}{1{,}54\ N/mm^2} = 0{,}15\ \rightarrow 15\%$$

3.4.3. Conclusion from shear design

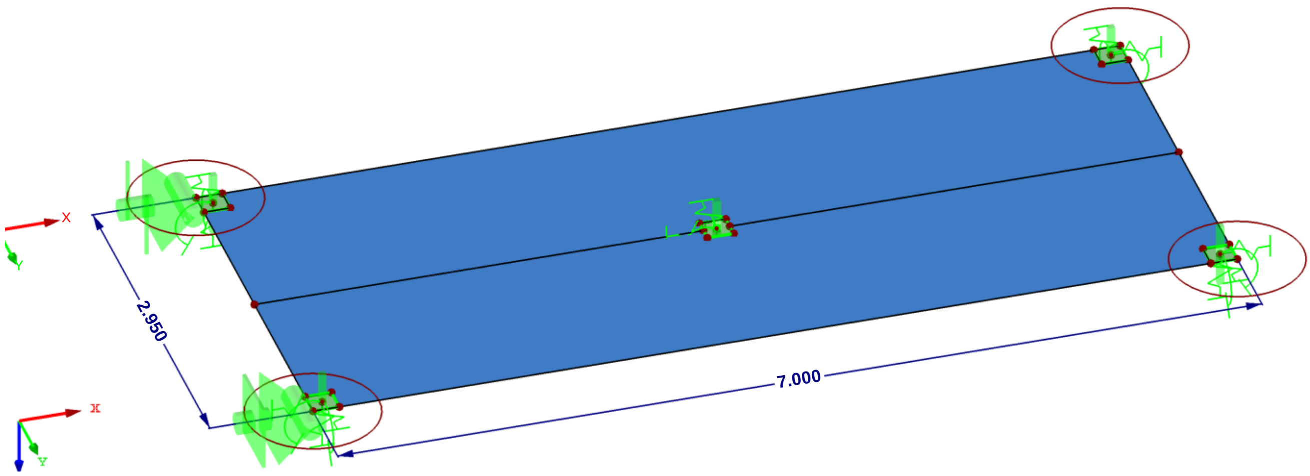
In both cases, at the center support and at the corner supports, the design bearing pressure does not exceed the design compressive strength of C24 timber, perpendicular to the grain (cross grain bearing) → o.k.

4. Analysis sample - reinforced

A CLT floor slab shall be supported by 5 point supports – one at each corner of the CLT slab and one in the center of the slab. The ULS design shall be performed for the given support condition.
This are the same assumptions as in the case above, just with increased loads, so reinforcement is required.

4.1. Geometry & load assumption

The following image has been taken from the finite element software RFEM by Dlubal.



Geometry & material:

- CLT 200 L5s, pine C24
- Length l = 7,00 m
- Width w = 2,95 m
- End plates of the steel columns are all 200 x 200 mm. These end plates are the supports for the given slab.

Loading:

- Dead load: self-weight of the CLT slab + a surcharge of 2,6 kN/m²
- Live load: 5,00 kN/m²

4.2. Support reactions by FE software

The support reactions have been taken from the finite element analysis software:

Center support:

- DL = 41,17 kN
- LL = 57,19 kN

Design support reaction (governing load combination):

$$P_d = \sum \gamma_{G,J} \cdot G_{k.j} + \gamma_{Q,1} \cdot Q_{k.1} \cdot \sum \gamma_{Q,i} \cdot \psi_{0,i} \cdot Q_{k.i}$$

$$P_{center,d} = 1{,}35 \cdot DL + 1{,}50 \cdot LL = 1{,}35 \cdot 41{,}17\ kN + 1{,}50 \cdot 57{,}19\ kN = \mathbf{141{,}36\ kN}$$

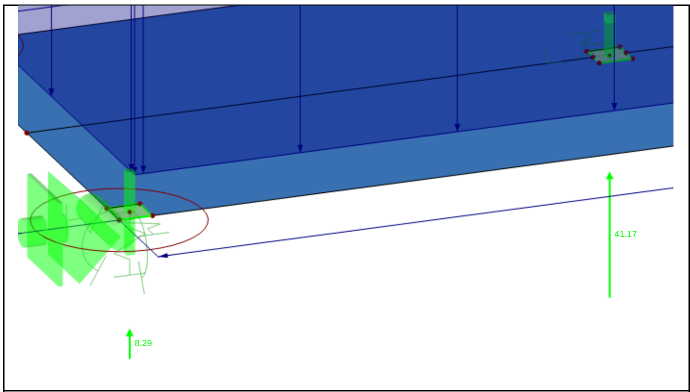
Corner supports:

DL = 8,29 kN

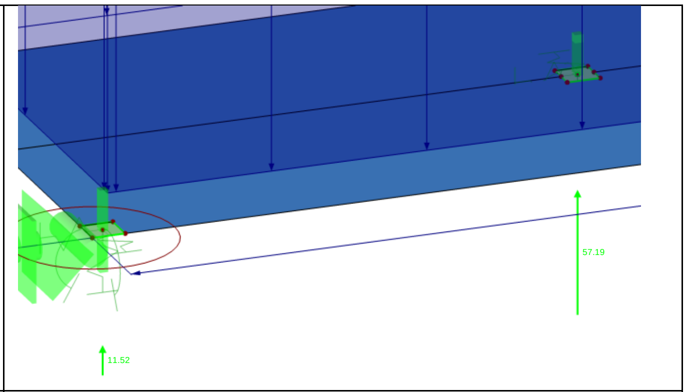
LL = 11,52 kN

Design support reaction (governing load combination):

$$P_{corner,d} = 1{,}35 \cdot DL + 1{,}50 \cdot LL = 1{,}35 \cdot 8{,}29\ kN + 1{,}50 \cdot 11{,}52\ kN = \mathbf{28{,}47\ kN}$$



Screenshot RFEM: DL support reaction



Screenshot RFEM: LL support reaction

4.3. Shear design for point support

4.3.1. Center support

Shear analysis for the unreinforced section.

$$V_{xz,d} = 0{,}33 \cdot n^{-0{,}1} \cdot P_{center,d} = 0{,}33 \cdot 5^{-0{,}1} \cdot 141{,}360\ kN = 39{,}72\ kN$$

$$V_{yz,d} = 0{,}5 \cdot P_{center,d} - V_{xz,d} = 0{,}5 \cdot 141{,}36 - 39{,}72 = 30{,}96\ kN$$

$$b_{ef,x} = b_{ef,y} = b_{A,x} + d \cdot tan35° = 200\ mm\ + 200\ mm\ \cdot tan35° = 340\ mm$$

$$\tau_{R,xz,d} = \frac{V_{xz,d}}{b_{ef,x} \cdot k_{R,x} \cdot (d_x + d_y)} = \frac{39.720\ N}{340\ mm\ \cdot 2\ \cdot (40+40)} = 0{,}73\ N/mm^2 (not\ governing)$$

$$\tau_{R,yz,d} = \frac{V_{yz,d}}{b_{ef,y} \cdot k_{R,y} \cdot (d_x + d_y)} = \frac{30.960\ N}{340\ mm\ \cdot 1\ \cdot (40+40)} = 1{,}14 N/mm^2\ \ (governing)$$

$$f_{R,k,pine} = min\begin{cases} 1{,}70 \\ 1{,}90 - \dfrac{t_{max,cross\ layer}[mm]}{100} \end{cases} \rightarrow = 1{,}90 - \frac{40\ mm}{100} = 1{,}50\ N/mm^2$$

$$\tau_{R,xz,d} = 0{,}73\ N/mm^2 < f_{R,d} = \frac{f_{R,k} \cdot k_{mod}}{\gamma_M} = \frac{1{,}50\ N/mm^2 \cdot 0{,}80}{1{,}30} = 0{,}92\ N/mm^2 \rightarrow\ o.k.\ ✓$$

$$Utilization\ rate\ \ UR = \frac{0{,}73\ N/mm^2}{0{,}92\ N/mm^2} = 0{,}79\ \ \rightarrow 79\%$$

$$\tau_{R,yz,d} = 1{,}14\ N/mm^2 > f_{R,d} = \frac{f_{R,k} \cdot k_{mod}}{\gamma_M} = \frac{1{,}50\ N/mm^2 \cdot 0{,}80}{1{,}30} = 0{,}92\ N/mm^2 \rightarrow\ not\ o.k.$$

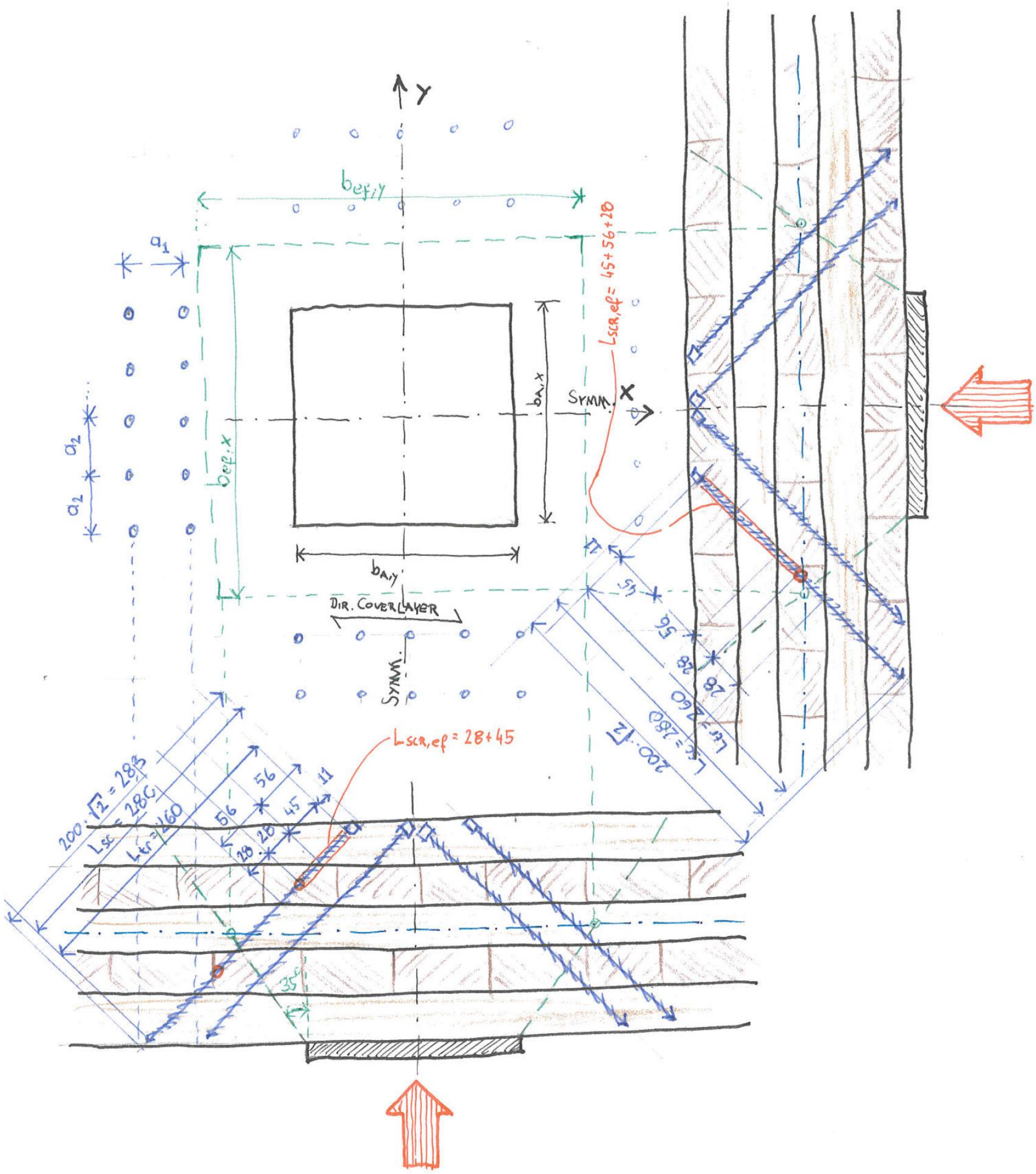
$$Utilization\ rate\ \ UR = \frac{1{,}14\ N/mm^2}{0{,}92\ N/mm^2} = 1{,}24\ \ \rightarrow 124\%$$

The design rolling shear at the central support is exceeding the design rolling shear strength in y direction. Therefore the section shall be strengthened at the support by fully threaded screws.

Use Rothoblaas VGZ fully threaded screws 9x280 (L_{scr} = 280 mm, L_{tr} = 260 mm); inclination 45°

The layout for the strengthening is displayed on the sketch below.

System sketch for strengthening:

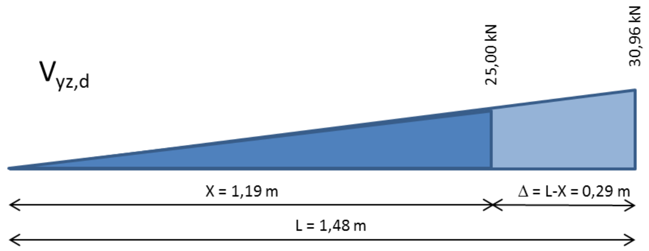


Dimensions, related to the sketch of the strengthened system:

Apply strengthening equally in all directions.

a_1 = 50 mm (chosen)

Number of screws in a_1 direction:
Screws are required in that part of the CLT section, where the design shear strength is exceeding the design shear stress, so strengthening is required. To determine that width Δ (see sketch), the shear distribution from the support towards the edge of the panels was assumed to be linear (conservative). Actually the strengthening would be only required along the y-axis ($\tau_{R,yz,d}$ is exceeding the limit).



Number or rows in a1 direction = (290 mm / 50mm) +1 = 6 rows

a_2 = 50 mm (chosen)
Number of screw rows in a_2 direction = 5 (chosen)

$$a_{2,ef} = max \begin{cases} & a_2 = 50\ mm \\ b_{ef}/n_{\perp rows} = & 340\ mm/5\ rows = 68\ mm \end{cases} \rightarrow a_{2,ef} = 68\ mm$$

$$b_{ef,x} = b_{ef,y} = b_{A,x} + d \cdot tan35° = 200\ mm + 200\ mm \cdot tan35° = 340\ mm$$

Design equation for the strengthened system:

$$\tau_{R,d} \le \bar{f}_{R,d} = \frac{k_{mod} \cdot \bar{f}_{R,k}}{\gamma_M}$$

$$\bar{f}_{R,k} = k_{R,90} \cdot f_{R,k} + \frac{R_{ax,k}}{\sqrt{2} \cdot a_1 \cdot a_{2,ef}}$$

Characteristic axial strength of a fully threaded screw Rothoblaas VGZ 9x280:

Characteristic tensile strength of the screw:

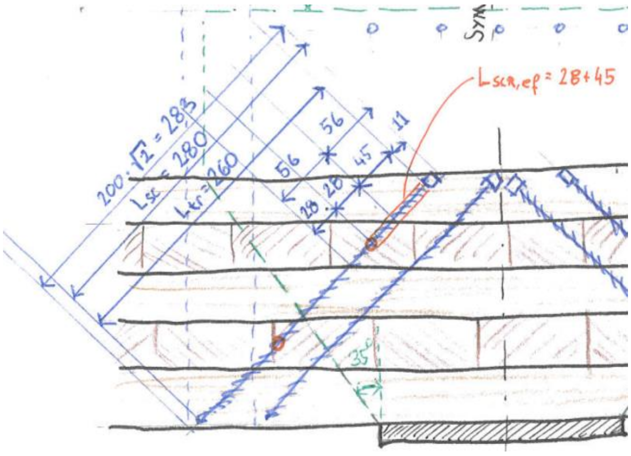
$$F_{tens,k} = 25.400\ N$$

Characteristic axial pull-out strength of the screw in x-direction (screw inclined in x-direction):

Embedment length:

$$L_{scr,ef,45°} = \overbrace{\underbrace{40\ mm \cdot \sqrt{2}}_{=56mm} - \underbrace{11\ mm}_{L_{Head}}}^{Embedment\ in\ layer\ 1} = 45\ mm$$

$$L_{scr,ef,90°} = \overbrace{\underbrace{40\ mm \cdot \sqrt{2}}_{=56mm} \cdot \underbrace{0{,}5}_{only\ until\ middle\ of\ layer\ 2}}^{Embedment\ in\ layer\ 2} = 28\ mm$$



$$L_{scr,ef} = 45\ mm + 28\ mm = 73\ mm$$

$$R_{ax,x,k} = 11{,}7 \cdot d \cdot \left(\frac{l_{ef,L}}{1{,}2 \cdot cos^2\alpha_L + sin^2\alpha_L} + \frac{l_{ef,C}}{1{,}2 \cdot cos^2\alpha_C + sin^2\alpha_C}\right) =$$

$$= 11{,}7 \cdot 9mm \cdot \left(\frac{45\ mm}{1{,}2 \cdot cos^2 45° + sin^2 45°} + \frac{28\ mm}{1{,}2 \cdot cos^2 90° + sin^2 90°}\right) = 7.256\ N$$

The pull-out strength of the screw in x-direction (7.256 N) is less and therefore design governing in x direction over the characteristic tensile strength of the screw (25.400 N).

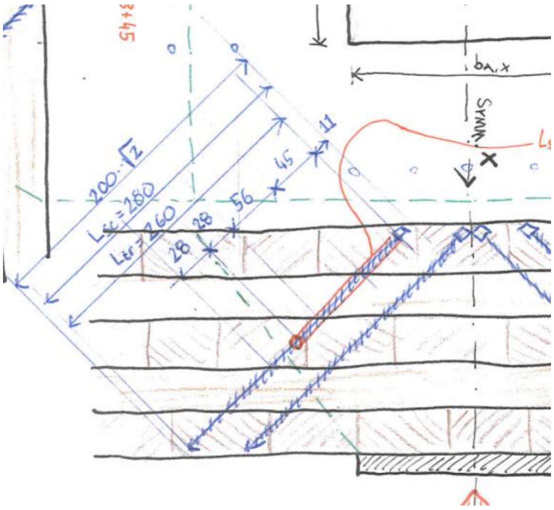
Characteristic axial pull-out strength of the screw in y-direction (screw inclined in y-direction):

Embedment length:

$$L_{scr,ef,90°} = \overbrace{\underbrace{40\ mm \cdot \sqrt{2}}_{=56mm} - \underbrace{11\ mm}_{L_{Head}}}^{Embedment\ in\ layer\ 1} + \overbrace{\underbrace{40\ mm \cdot \sqrt{2}}_{=56mm} \cdot \underbrace{0{,}5}_{only\ until\ middle\ of\ layer\ 2}}^{Embedment\ in\ layer\ 3}$$

$$= 73\ mm$$

$$L_{scr,ef,45°} = \overbrace{\underbrace{40\ mm \cdot \sqrt{2}}_{=56mm} \cdot}^{Embedment\ in\ layer\ 2} = 56\ mm$$



$$L_{scr,ef} = 73\ mm + 56\ mm = 129\ mm$$

$$R_{ax,y,k} = 11{,}7 \cdot d \cdot \left(\frac{l_{ef,L}}{1{,}2 \cdot cos^2\alpha_L + sin^2\alpha_L} + \frac{l_{ef,C}}{1{,}2 \cdot cos^2\alpha_C + sin^2\alpha_C} \right) =$$

$$= 11{,}7 \cdot 9mm \cdot \left(\frac{73\ mm +}{1{,}2 \cdot cos^2 90° + sin^2 90°} + \frac{56\ mm}{1{,}2 \cdot cos^2 45° + sin^2 45°} \right) = 13.048\ N$$

The pull-out strength of the screw in y-direction (13.048 N) is less and therefore design governing in y direction over the characteristic tensile strength of the screw (25.400 N).

Rolling shear verification in x direction:

$$\sigma_{c,90,x,k} = \frac{F_k}{b_{ef,x} \cdot b_{ef,y}} + \frac{R_{ax,x,k}/\sqrt{2}}{a_1 \cdot a_{2,ef}} = \frac{41170\ N + 57190\ N}{340\ mm \cdot 340\ mm} + \frac{7256\ N/\sqrt{2}}{50\ mm \cdot 68\ mm} = 2{,}35\ N/mm^2$$

$$k_{R,90,x} = \min \begin{cases} 1 + 0{,}35 \cdot \sigma_{c,90,x,k} = 1 + 0{,}35 \cdot 2{,}35\ N/mm^2 = 1{,}83 \\ 1{,}20 \end{cases} \rightarrow = 1{,}20$$

$$\bar{f}_{R,x,k} = k_{R,90} \cdot f_{R,k} + \frac{R_{ax,x,k}}{\sqrt{2} \cdot a_1 \cdot a_{2,ef}} = 1{,}20 \cdot 1{,}50\ N/mm^2 + \frac{7.256\ N}{\sqrt{2} \cdot 50\ mm \cdot 68\ mm} = 3{,}31\ N/mm^2$$

$$\tau_{R,xz,d} = 0{,}73\ N/mm^2 < \frac{k_{mod} \cdot \bar{f}_{R,x,k}}{\gamma_M} = \frac{0{,}8 \cdot 3{,}31\ N/mm^2}{1{,}30} = 2{,}04\ N/mm^2 \rightarrow o.k.\ \checkmark$$

$$Utilization\ rate\ \ UR = \frac{0{,}73\ N/mm^2}{2{,}04\ N/mm^2} = 0{,}36\ \rightarrow 36\%$$

Rolling shear verification in y direction:

$$\sigma_{c,90,y,k} = \frac{F_k}{b_{ef,x} \cdot b_{ef,y}} + \frac{R_{ax,y,k}/\sqrt{2}}{a_1 \cdot a_{2,ef}} = \frac{41170\ N + 57190\ N}{340\ mm \cdot 340\ mm} + \frac{13.048\ N/\sqrt{2}}{50\ mm \cdot 68\ mm} = 3{,}56\ N/mm^2$$

$$k_{R,90,y} = \min \begin{cases} 1 + 0{,}35 \cdot \sigma_{c,90,y,k} = 1 + 0{,}35 \cdot 4{,}50\ N/mm^2 = 2{,}25 \\ 1{,}20 \end{cases} \rightarrow = 1{,}20$$

$$\bar{f}_{R,y,k} = k_{R,90} \cdot f_{R,k} + \frac{R_{ax,y,k}}{\sqrt{2} \cdot a_1 \cdot a_{2,ef}} = 1{,}20 \cdot 1{,}50\ N/mm^2 + \frac{13.048\ N}{\sqrt{2} \cdot 50\ mm \cdot 68\ mm} = 4{,}50\ N/mm^2$$

$$\tau_{R,xz,d} = 1{,}14\ N/mm^2 < \frac{k_{mod} \cdot \bar{f}_{R,x,k}}{\gamma_M} = \frac{0{,}8 \cdot 4{,}50\ N/mm^2}{1{,}30} = 2{,}77\ N/mm^2 \rightarrow o.k.\ \checkmark$$

$$Utilization\ rate\ \ UR = \frac{1{,}14\ N/mm^2}{2{,}77\ N/mm^2} = 0{,}41\ \rightarrow 41\%$$

Due to the low utilization rates in both directions, the screw spacing in a1 direction could be extended. This design step will not be executed in this document, but it shall be mentioned, that this would be possible and reasonable.

4.3.2. Corner support

The shear analysis for the corner support is analogous to the analysis for the central support and will be skipped in this document.

4.4. Bearing pressure analysis

4.4.1. Center support - unreinforced

$$\sigma_{c,90,d} = \frac{P_{center,d}}{k_{c,90} \cdot A_{eff}} = \frac{141.360\ kN}{1{,}9 \cdot [200\ mm\ \cdot (2 \cdot 30\ mm + 200\ mm)]} = 1{,}43\ N/mm^2$$

$$f_{c,90,d} = \frac{f_{c,90,k} \cdot k_{mod}}{\gamma_M} = \frac{2{,}5\ N/mm^2 \cdot 0{,}80}{1{,}30} = 1{,}54\ N/mm^2$$

$$\sigma_{c,90,d} = 1{,}43\ N/mm^2 \leq f_{c,90,d} = 1{,}54 N/mm^2 \rightarrow o.k. \checkmark$$

$$Utilization\ rate\ \ UR = \frac{0{,}74\ N/mm^2}{1{,}54\ N/mm^2} = 0{,}92\ \ \rightarrow 92\%$$

With 92% utilization, the bearing pressure analysis would actually pass the design criterion. For demonstrative purposes the strengthening of the bearing section shall be executed.

4.4.2. Center support - reinforced

The reinforcement at a bearing plate shall be done with fully threaded screws. The screw heads shall be flush with the bottom surface of the CLT, so all screws receive load from the bearing plate. Rothoblaas VGS 9x160 shall be used.

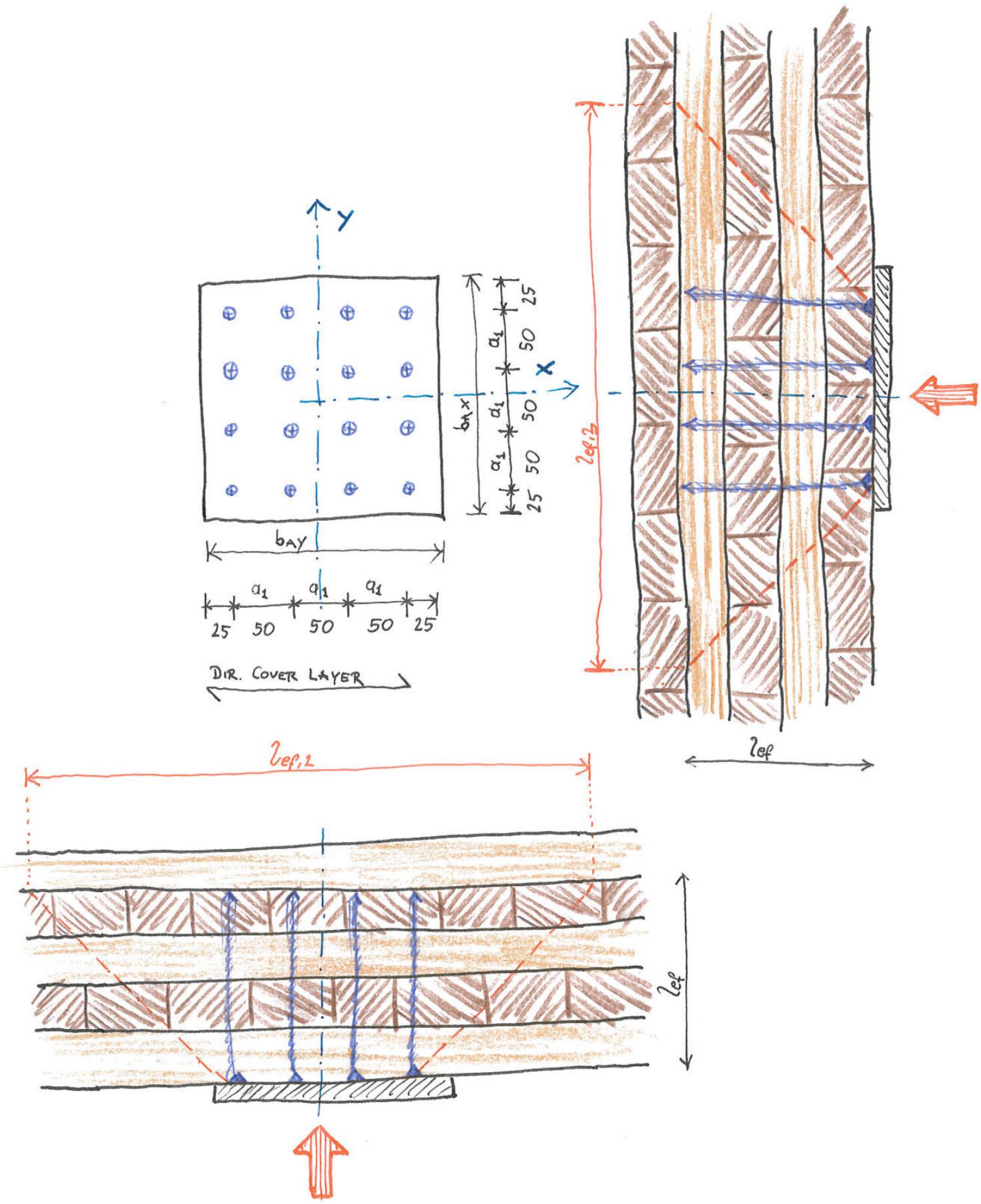
Geometry:

Bearing plate dimensions:
b_{Ax} = 200 mm
b_{Ay} = 200 mm

Screw spacing a_1 = 50 mm
Number of screws: 4 x 4 = 16 screws

Screw length L_{scr} = 160 mm
Thread length L_{tr} = 150 mm

Systems ketch for strengthening at bearing plate



Point support of CLT

The design equation for the characteristic load bearing resistance of the CLT perpendicular to the grain is as follows:

$$F_{90,Rk} = min \begin{cases} k_{c,90} \cdot A_{eff} \cdot f_{c,90,k} + n \cdot min \left\{ \overbrace{F_{ax,Rk}}^{axial\ withdrawal\ strength} ; \underbrace{F_{ki,Rk}}_{buckling\ strength} \right\} \\ l_{ef,3} \cdot l_{ef,2} \cdot f_{c,90,k} \end{cases}$$

Axial withdrawal strength:

$$F_{ax,Rk} = 11{,}7 \cdot d \cdot \left(\frac{l_{ef,L}}{1{,}2 \cdot cos^2\alpha + sin^2\alpha}\right) = 11{,}7 \cdot 9\ mm \cdot \left(\frac{150\ mm}{1{,}2 \cdot cos^2 90° + sin^2 90°}\right) = 15.795\ N$$

Characteristic ideal elastic buckling load:

Elastic embedment of the screw:

$$c_h = (0{,}19 + 0{,}012 \cdot d) \cdot \rho_k \cdot \left(\frac{\alpha}{180°} + 0{,}5\right) =$$

$$= (0{,}19 + 0{,}012 \cdot 9) \cdot 350 \cdot \left(\frac{90}{180°} + 0{,}5\right) = 104{,}3\ N/mm^2$$

Second moment of inertia:

$$I_s = \frac{\pi}{64} \cdot {d_1}^4 = \frac{\pi}{64} \cdot 9^4 = 322\ mm^4$$

Modulus of elasticity: E_s = 210.000 N/mm²

Characteristic ideal elastic buckling load:

$$N_{ki,k} = \sqrt{c_h \cdot E_s \cdot I_s}$$
$$= \sqrt{322\ mm^4 \cdot 210.000\ N/mm^2 \cdot 104{,}3 N/mm^2} = \mathbf{83.988\ N}$$

Relative slenderness ratio:

Characteristic yield strength for carbon steel
$f_{y,k}$ = 1.000 N/mm²

Characteristic axial plastic strength of the screw:

$$N_{pl,k} = \pi \cdot \frac{{d_1}^2}{4} \cdot f_{y,k} = \pi \cdot \frac{9^2}{4} \cdot 1.000\ N/mm^2 = 63.617\ N$$

Relative slenderness ratio:

$$\bar{\lambda}_k = \sqrt{\frac{N_{pl,k}}{N_{ki,k}}} = \sqrt{\frac{63.617\ N}{83.988\ N}} = \mathbf{0{,}87}$$

storaenso rethink.

Characteristic buckling capacity:

$$k = 0{,}5 \cdot \left[1 + 0{,}49 \cdot (\bar{\lambda}_k - 0{,}2) + \bar{\lambda}_k^{\,2}\right] =$$

$$= 0{,}5 \cdot [1 + 0{,}49 \cdot (0{,}87 - 0{,}2) + 0{,}87^2] = 1{,}04$$

$$\kappa_c = \begin{cases} 1 & for\ \bar{\lambda}_k \leq 0{,}2 \\ \dfrac{1}{k + \sqrt{k^2 - \bar{\lambda}_k^{\,2}}} & for\ \bar{\lambda}_k > 0{,}2 \end{cases}$$

$$\bar{\lambda}_k = 0{,}87 > 0{,}2$$

$$\kappa_c = \frac{1}{k + \sqrt{k^2 - \bar{\lambda}_k^{\,2}}} = \frac{1}{1{,}04 + \sqrt{1{,}04^2 - 0{,}87^2}} = 0{,}62$$

Characteristic buckling capacity:

$$F_{ki,Rk} = \kappa_c \cdot N_{pl,k} = 0{,}62 \cdot 63.617 = \mathbf{39.443\ N}$$

Characteristic load bearing resistance of the CLT perpendicular to the grain:

$$F_{90,Rk} = min \begin{cases} k_{c,90} \cdot A_{eff} \cdot f_{c,90,k} + n \cdot min \begin{Bmatrix} \overbrace{F_{ax,Rk}}^{axial\ withdrawal\ strength} & ; & \underbrace{F_{ki,Rk}}_{buckling\ strength} \end{Bmatrix} \\ l_{ef,3} \cdot l_{ef,2} \cdot f_{c,90,k} \end{cases}$$

$$= min \begin{cases} 1{,}90 \cdot [200\ mm \cdot (30\ mm + 200\ mm + 30\ mm)] \cdot 2{,}5\ N/mm^2 + 16 \cdot min \begin{Bmatrix} \overbrace{\mathbf{15.795\ N}}^{axial\ withdrawal\ strength} & ; & \underbrace{39.443\ N}_{buckling\ strength} \end{Bmatrix} \\ (160\ mm + 3 \cdot 50\ mm + 160\ mm) \cdot (160\ mm + 3 \cdot 50\ mm + 160\ mm) \cdot 2{,}5\ N/mm^2 \end{cases}$$

$$= min \begin{cases} 247.000\ N + 252.720\ N = \mathbf{499.720\ N} \\ 552.250\ N/mm^2 \end{cases}$$

$$F_{90,Rk} = 499.720\ N$$

Design load bearing resistance of the central support:

$$F_{90,Rd} = \frac{F_{90,Rk} \cdot k_{mod}}{\gamma_M} = \frac{449.720\ N \cdot 0{,}80}{1{,}30} = 276.750\ N$$

$$\sigma_{c,90,d} = \frac{P_{center,d}}{k_{c,90} \cdot A_{eff}} = \frac{141.360\ kN}{1{,}9 \cdot [200\ mm \cdot (2 \cdot 30\ mm + 200\ mm)]} = 1{,}43\ N/mm^2$$

$$P_{center,d} = 141.360\ N < F_{90,Rd} = 276.750\ N \rightarrow o.k. ✓$$

$$Utilization\ rate\ UR = \frac{141.360\ N}{276.750\ N} = 0{,}51 \rightarrow 51\%$$

4.4.3. Conclusion strengthening design

The support would be capable to withstand the applicable design pressure perpendicular to the grain with an utilization rate (UR) of 92%. With the help of 16 8x160 VGS screws (fully threaded), the utilization rate could be dropped to 51%. This is an increase in strength of 55%.

Thomas Demschner
R&D Engineer
Stora Enso Wood Products GmbH

Ybbs a.d. Donau, 22.01.2014

5. Bibliography

Blass, P.-I., & Görlacher, D.-I. (n.d.). *Compression perpendicular to the grain.* Universität Karlsruhe.

Bogensperger, D., Augustin, D., & Schickhofer, P. (n.d.). *Properties of CLT-Panels Exposed to Compression Perpenducular to their Plane.* Graz: Holzbau Forschungs GmbH & Institute for Timber Engineering and Wood Technology, TU Graz.

Mestek, D. P. (2011). *Punktegestützte Flächentragwerke aus Brettsperrholz (BSP) - Schubmessung unter Berücksichtigung von SChubverstärkungen.* München: Technische Universität München.

Rothoblaas. (2012). *European Technical Approval ETA-11/0030.* Charlottenlund: ETA-Danmark A/S.

Wallner-Novak, D. (2013). *Brettsperrholz Bemessung - Grundlagen für die Statik und Konstruktion nach Eurocode.* pro Holz.